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SUBJECT AREA: FINANCE AND ECONOMICS

Multivariate Models to Forecast Portfolio Value at Risk: from the Dot-Com crisis to the global financial crisis

Modelos Multivariados na Previsão do Valor em Risco de Carteiras de Investimento: da crise das empresas tecnológicas à crise financeira global

Modelos multivariados en la previsión del valor en riesgo de carteras de inversión: de la crisis de las empresas tecnológicas a la crisis financiera global

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ABSTRACT

This study analyzed market risk of an international investment portfolio by means of a new methodological proposal based on Value-at-Risk, using the covariance matrix of multivariate GARCH-type models and the extreme value theory to realize if an international diversification strategy minimizes market risk, and to determine if the VaR methodology adequately captures market risk, by applying Backtesting tests. To this end, we considered twelve international stock indexes, accounting for about 62% of the world stock market capitalization, and chose the period from the Dot-Com crisis to the current global financial crisis. Results show that the proposed methodology is a good alternative to

accommodate the high market turbulence and can be considered as an adequate portfolio risk management instrument.

Keywords: Stock markets. Value at Risk. Multivariate GARCH models. Extreme value theory. Backtesting.

RESUMO

Neste estudo é analisado o risco de mercado de uma carteira de investimento internacional por meio de uma nova proposta metodológica, baseada no *Value-at-Risk*, recorrendo à matriz de covariâncias de modelos multivariados do

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tipo GARCH e à teoria dos valores extremos, para perceber se uma estratégia de diversificação internacional minimiza o risco de mercado, assim como para verificar se a metodologia VaR capta adequadamente esse mesmo risco, aplicando testes de validação de performance. Para o efeito, foram selecionados 12 índices bolsistas internacionais, representativos de cerca de 62% da capitalização bolsista mundial, e escolhido o período compreendido entre a crise Dot-Com e a atual crise financeira global. Os resultados obtidos mostram que a proposta metodológica é uma boa alternativa na acomodação da elevada turbulência dos mercados, podendo ser considerada como uma ferramenta válida na gestão do risco de carteiras de investimento.

Palavras-chave: Mercados bolsistas. *Value-at-Risk*. Modelos multivariados GARCH. Teoria dos valores extremos. *Backtesting*.

RESUMEN

Este estudio analiza el riesgo de mercado de una cartera de inversiones internacionales a través de una nueva propuesta metodológica en torno al valor en riesgo, usando la matriz de covarianza de modelos multivariados GARCH y la teoría de valores extremos, para ver si una estrategia de diversificación internacional minimiza el riesgo de mercado, así como para observar si la metodología VaR capta adecuadamente el riesgo de mercado, aplicando pruebas de validación. Con este fin, se consideraron doce índices bursátiles internacionales que representan aproximadamente el 62% de la capitalización bursátil mundial, eligiéndose el período entre las crisis Dot.com y financiera mundial. Los resultados muestran que la metodología propuesta puede ser utilizada para acomodar la alta turbulencia de los mercados, y que puede ser considerada como una herramienta válida en la gestión del riesgo de las carteras de inversión.

Palabras clave: Mercados de valores. Valor en riesgo. Modelos multivariados GARCH. Teoría de los valores extremos. *Backtesting*.

1 INTRODUCTION

The quantification and risk diversification have drawn attention from researchers and market professionals. The publication, in 1952, of Harry Markowitz's article titled "*Portfolio Selection*" gave rise to the portfolio theory (MARKOWITZ, 1952). This theory allows the selection of optimal portfolios, under the assumption that rational and risk-averse investors want to maximize portfolio profitability, compared with a given level of risk. Markowitz showed that investors can minimize risk (variance) if they select assets that do not have identical movements, i.e., if they include in their portfolio assets that are not highly correlated. The proposal made by Markowitz, of considering the variance of financial assets returns as a measure of risk, was universal until the end of the 1980s. Several different market situations, as the stock market crash in 1987, the bankruptcy of the Barrings Bank, increased volatility in stock markets and investment in emerging markets, among others, increased the need to develop risk measures that express potential losses.

The signing of the Basel Accord in 1988, strengthened the purpose of standardizing evaluation standards of financial institutions soundness and establishing mandatory minimum levels of capital, depending on exposure to credit risk of each institution, in order to minimize banking risks and to enable comparative analysis of international banking institutions. In 1996, a second accord was signed, known as Basel II, whose central objective was the issue of market risk, and that required that financial institutions maintain certain levels of prudential capital in order to absorb potential losses and avoid bankruptcy, also ensuring economic stability (MANGANELLI; ENGLE, 2001). With this new accord, financial institutions were authorized to adopt internal models for assessing financial risks and calculating equity, by means of validation and control by the supervisory authorities. A new accord, which became known as Basel III, which intended to redesign the banking system and the largest credit institutions, cited as the main responsible for the crisis that began in 2007, was

signed in 2010. This accord introduced some changes to the second accord, particularly in relation to the increase in capital requirements of banks and the introduction of liquidity and leverage standards.

The need of investors and market professionals having risk measures available, which would translate the probability of potential losses on investments, led to new approaches to risk management. Modern risk management has taken a significant step with the presentation of the Value-at-Risk (VaR) methodology in 1993, and the RiskMetrics methodology in 1994. In early 1996, the Basel Committee would consider using the VaR methodology to measure market risk, and authorized the banks to use internal models to estimate it. This methodology has revolutionized risk management, became a kind of benchmark in the analysis and management of market risk, and provided an estimate of the maximum potential loss that investors may incur, depending on the total exposure of their investment positions (JORION, 2007).

Most studies assume that assume VaR estimates use conditional heteroskedasticity models, commonly known as the ARCH models, as a result of the work of Engle (1982), which hold the effects of volatility clustering and asymmetry. So and Yu (2006), Wu and Shieh (2007) and Niguez (2008) applied this approach considering univariate analyses.

The multivariate models of conditional heteroskedasticity are presented in the literature as an alternative market risk analysis of investment portfolios (FERREIRA; LOPEZ, 2005). Its use, however, has been clearly less frequent than that of univariate models. Of the works using multivariate analyzes, we point out the work of Morimoto and Kawasaki (2008) and Caporin and McAleer (2012). The first one compared the performance of various multivariate GARCH-type models, namely VECH, BEKK, CCC and the DCC, for the normal and t-student distributions, in forecasting the VaR of a portfolio investment, comprising a wide range of assets of the Tokyo Stock Exchange, and concluded that the GARCH-DCC model showed the best

performance in the VaR forecast. The authors of the second work also resorted to several multivariate GARCH-type models to conclude that the performance of the models depends on the sample period and the type of portfolio.

According to several authors, including McNeil and Frey (2000), Marimoutou, Raggad, Trabelsi (2009), Assaf (2009) and Andreev et al. (2009), the Extreme Value Theory (EVT) shows good ability to accommodate the occurrence of extreme observations. However, studies that consider, at the same level, the multivariate GARCH-type models and EVT, to accommodate situations of high turbulence, remain unknown.

In this research, we intend to expand the existing finance literature, in empirical and methodological terms, using various multivariate models of conditional heteroskedasticity and EVT, to estimate VaR, in order to more appropriately accommodate situations of high market turbulence, in particular those experienced during Dot-Com and Global Financial crises. On the other hand, and differently compared to the referred to work, which included a set of assets in a particular market, in this paper, we consider an internationally diversified investment portfolio, consisting of indexes representing various geographies and levels of development, particularly indexes representing European states under international financial assistance, in order to form a strong conclusion on the ability of these models in risk management of international investment portfolios, in a context of high volatility and turbulence.

In order to estimate VaR, we will consider the Delta-Normal, GARCH-VECH, GARCH-CCC and GARCH-BEKK approaches, assuming normal distribution and the t-Student distribution. In addition, these models will be considered, including, on the one hand, the assumption of the asymmetric effect, by specifying Threshold GARCH (TGARCH) and, on the other, EVT.

This study follows the following structure: part 2 presents the methodology chosen; part 3 presents information about the data and the empirical results, whereas part 4 briefly presents our main conclusions.

2 METHODOLOGY

2.1 Value at risk

According to Best (1998), Value at Risk is the maximum amount that is expected to be lost in an asset or a portfolio of assets during a given period and for a given confidence level.

Formally, the VaR can be defined as follows:

$$P[\epsilon L_t > \epsilon VaR_t] = \alpha \tag{1}$$

in which $(1 - \alpha)$ is the confidence level and L is the loss, i.e., the change in value of the portfolio.

The VaR can also be defined in terms of the rate distribution geometric return of the portfolio. Considering the probability α and assuming that the returns of an asset or a portfolio, R_{PF} , follow normal distribution with mean zero and standard deviation $\sigma_{PF,t+1}$, we have $VaR_{t+1}^\alpha = -\sigma_{PF,t+1} * \Phi_\alpha^{-1}$.

According to Jorion (2007), Tsay (2005) and Esch, Kieffer, Lopez (2005), the t-student distribution gathers, however, greater consensus to describe the behavior of financial assets. The

superiority of this distribution has been reported by several authors in different contexts and in many empirical studies (ANGELIDIS, BENOS, DEGIANNAKIS, 2004; GIOT, LAURENT, 2003). When the returns of a portfolio of assets are described by the t-Student distribution, the VaR is given by (CHRISTOFFERSEN, 2003):

$$VaR_{t+1}^\alpha = -\sigma_{PF,t+1} \sqrt{\frac{d-2}{d}} t_\alpha^{-1}(d) \tag{2}$$

in which $t_\alpha^{-1}(d)$ is the quantile to the left of α , of the t-student distribution, with d degrees of freedom.

Portfolio profitability, $R_{PF,t+1}$, on day $t+1$, is determined considering the following equation:

$$R_{PF,t+1} = \sum_{i=1}^n w_i R_{i,t+1} \tag{3}$$

in which $R_{i,t+1}$ is the return related to index i , on day $t+1$, n is the number of indexes comprising the portfolio and w_i is the weight assigned to index i . Accordingly, portfolio return is the function of relative weights assigned to indexes and of the returns of each index.

Portfolio variance is:

$$\sigma_{PF,t+1}^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij,t+1} = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij,t+1} \sigma_{j,t+1} \rho_{ij,t+1} \tag{4}$$

in which $\sigma_{ij,t+1}$ and $\rho_{ij,t+1}$ are the covariance and correlation, respectively, between asset i and j , on day $t+1$. Considering that $\sigma_{ij,t+1} = \sigma_{ji,t+1}$ and that $\rho_{ij,t+1} = \rho_{ji,t+1}$, for all i and j , and that $\rho_{ii,t+1} = 1$ and $\rho_{ij,t+1} = \rho_{ji,t+1}$, for all i and j , with $\rho_{ii,t+1} = 1$ and $\sigma_{ii,t+1} = \sigma_{i,t+1}^2$, for all i , we have:

$$\sigma_{PF,t+1}^2 = W' \sum_{t+1} W \tag{5}$$

In the previous expression, W is the vector of the portfolio weights and \sum_{t+1} is the covariance matrix of returns. In the case of $n=12$, we have:

$$\sigma_{PF,t+1}^2 = [w_1 \ w_2 \ \dots \ w_{12}] \times \begin{bmatrix} \sigma_{1,t+1}^2 & \sigma_{1,2,t+1} & \dots & \sigma_{1,12,t+1} \\ \sigma_{2,1,t+1} & \sigma_{2,t+1}^2 & \dots & \sigma_{2,12,t+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{12,1,t+1} & \sigma_{12,2,t+1} & \dots & \sigma_{12,12,t+1} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{12} \end{bmatrix} \tag{6}$$

Regarding the weighting of the indexes, in this work we considered that they are constant over time and shared equally, whereby the relative weight of each index corresponds to 1/2 of total portfolio.

When we assume the normality of returns, the methodology presented above is called Delta Normal, and the VaR of the portfolio is given by:

$$VaR_{t+1}^\alpha = -\sigma_{PF,t+1} * \Phi_\alpha^{-1} \tag{7}$$

In order to incorporate conditional heteroskedasticity, usually present in the indexes of stock markets, variance-covariance matrixes are

$$vech(H_t) = C + \sum_{j=1}^q A_j vech(\epsilon_{t-1} \epsilon'_{t-1}) + \sum_{j=1}^p B_j vech(H_{t-j}) \tag{8}$$

in which H_t is related to the conditional variance and covariance matrix. According to Scherrer and Ribarits (2007), the understanding of the matrix becomes easier when it includes more than two variables, as in the case studied. The representation of the VECH model is based on the assumption that the conditional variance depends on the square of lagged residuals and the conditional covariance depends on the lagged cross-residuals and lagged covariance of other series (HARRIS, SOLLIS, 2003). In the model equation, A and B are coefficient matrixes, of size $\frac{1}{2} N(N+1) \times \frac{1}{2} N(N+1)$, C is a vector of constant terms, of size $\frac{1}{2} N(N+1) \times 1$, and p and q indicate the order number of GARCH and ARCH models, respectively.

Constant Conditional Correlation (CCC) model

The CCC model was proposed by Bollerslev (1990), and implied that the conditional correlation assumption is constant for the period t . The model covariance matrix is given by:

$$H_t = D_t R D_t \tag{9}$$

estimated, using various multivariate GARCH models, namely the VECH, BEKK and CCC models, whose theoretical assumptions are presented in the following section.

2.2 Conditional heteroskedasticity multivariate models

VECH Model

The GARCH-VECH model was proposed by Bollerslev, Engle, Wooldridge (1988), and can be written as follows:

in which H_t is the covariance matrix, D_t is a diagonal matrix, whose elements comprise the conditional variance, $h_{ij,t}$ of each series, at the time t , and R is the coefficient matrix of constant linear correlation, $p_{ij,t}$.

Assuming the conditional normal distribution, the constant linear correlation coefficient between the series i and j is given by:

$$\hat{p}_{ij} = \sum_{t=0}^T \hat{e}_{i,t} \hat{e}_{j,t} \left(\sum_{t=0}^T \hat{e}_{i,t}^2 \right)^{-1/2} \left(\sum_{t=0}^T \hat{e}_{j,t}^2 \right)^{-1/2} \tag{10}$$

in which $\hat{e}_{i,t}$ and $\hat{e}_{j,t}$ are the standardized residuals of series i and j , obtained by means of univariate estimate, using GARCH models. To obtain the covariance matrix, univariate models are estimated and standardized residuals are computed, which in turn serve as the basis for calculating linear correlation coefficients.

One way to measure the correlation involves processing the residuals as in the variance-covariance model. By definition,

$$\sigma_{ij,t+1} = \sigma_{i,t+1} \sigma_{j,t+1} \rho_{ij,t+1} \quad (11)$$

Accordingly,

$$\rho_{ij,t+1} = \frac{\sigma_{ij,t+1}}{\sigma_{i,t+1} \sigma_{j,t+1}} \quad (12)$$

Therefore, if, for instance, $\rho_{ij,t+1} = (1-\lambda)R_{i,t}R_{j,t} + \lambda\sigma_{ij,t}$, for any i,j , then:

$$\rho_{ij,t+1} = \frac{(1-\lambda)R_{i,t}R_{j,t} + \lambda\sigma_{ij,t}}{\sqrt{((1-\lambda)R_{i,t}^2 + \lambda\sigma_{i,t}^2)((1-\lambda)R_{j,t}^2 + \lambda\sigma_{j,t}^2)}} \quad (13)$$

BEKK model

An alternative estimate of the conditional variance is BEKK (-BABA-ENGLE-KRAFT-KRONER) model, proposed by Engle and Kroner (1995), which may be expressed as follows:

$$H_t = C'C + \sum_{j=1}^q A'_j \varepsilon_{t-1} \varepsilon'_{t-1} A_j + \sum_{j=1}^p B'_j H_{t-j} B_j \quad (14)$$

The BEKK model guarantees that matrix H_t is semi-positive, unlike what happens in the VECH model, in which there is no such guarantee. In the BEKK model, A and B are matrixes of size $N \times N$, C is an upper triangular matrix of coefficients, N is the number of series considered in the model, and p and q indicate the order number of GARCH and ARCH models, respectively.

2.3 Extreme value theory

One of the most recent methods of VaR calculation is based on the Extreme Value Theory (EVT). According to several authors, a great advantage of the EVT, vis-à-vis other approaches, is to provide good fit to the tails of distribution of returns (DANIELSSON, DE VRIES, 1997; EMBRECHTS, KLUPPERLBERG, MIKOSCH, 1997; MCNEIL, 1998; REISS, THOMAS, 1997).

One should consider that the probability of standardized return, z , reduced by the threshold, u , is less than a certain value x , taking

into account that the standardized return is above the threshold, u .

$$F_u(x) \equiv \Pr\{z - u \leq x | z > u\}, \text{ em que } x > u \quad (15)$$

The standardized returns above a threshold should also be considered, taking into account that the distribution, $F_u(x)$, depends on the choice of the threshold. Using the general definition of conditional probability, we have

$$F_u(x) = \frac{P_r\{u < z \leq x + u\}}{P_r\{z > u\}} = \frac{F(x + u) - F(u)}{1 - F(u)} \quad (16)$$

Thus, the distribution of standardized returns, above the threshold, can be written as a function of the distribution of standardized returns, $F(x)$.

Within the EVT, as the extreme values deviate from the threshold, u , converge to a generalized Pareto distribution (GPD), $G(x, \xi, \beta)$. This distribution is generally defined as follows:

$$G(x, \xi, \beta) = \begin{cases} 1 - (1 + \xi x / \beta)^{-1/\xi}, & \text{se } \xi \neq 0 \\ 1 - \exp(-x / \beta), & \text{se } \xi = 0 \end{cases} \quad (17)$$

With $\beta > 0$, and

$$\begin{cases} x \geq u, \text{ se } \xi \geq 0 \\ u \leq x \leq u - \beta/\xi, \text{ se } \xi < 0 \end{cases} \quad (18)$$

in which the coefficient of asymmetry, ξ , is positive and represents the rate of decay of the tail, β is the scale parameter and μ is the threshold.

Based on the methodology of McNeil (1999), considering the points x , with $x > u$, in the tail of the distribution. With $y = x + u$, we have:

$$F_u(x) = \frac{F(x + u) - F(u)}{1 - F(u)} \quad (19)$$



To obtain

$$F(y) = 1 - [1 - F(u)][1 - F_u(y - u)] \quad (20)$$

with T being the total sample size and T_u the number of observations above the threshold, u . The term $1 - F(u)$ can then be estimated as the ratio of observations (T_u/T) above the threshold. On the other hand, F_u^* may be obtained by the maximum likelihood estimator, the standardized observations, above a chosen threshold. Assuming that $\hat{1} \neq 0$, the distribution is

$$F(y) = 1 - T_u/T (1 + \xi(y - u)/\beta)^{-1/\xi} \quad (21)$$

The cumulative distribution function, equal to $1 - \alpha$, in which there is only one probability, α , of obtaining losses exceeding the standardized quantile value, is defined implicitly by:

$$F(F_{1-\alpha}^{-1}) = 1 - \alpha \quad (22)$$

Based on the definition of F^* , it can be resolved for the quantile so as to obtain:

$$F_{1-\alpha}^{-1} = \mu[\alpha/(T_u/T)]^{-\xi} \quad (23)$$

VaR is obtained from the EVT and the variance model chosen, based on the following expression:

$$VaR_{t+1}^p = \sigma_{PF,t+1} F_{1-p}^{-1} = \sigma_{PF,t+1} \mu[\alpha/(T_u/T)]^{-\xi} \quad (24)$$

in which α is the confidence level of VaR. To estimate the VaR with an α confidence level, we use the estimator EVT and a u cutoff point, so as to consider a data rate, of the left tail, exceeding $1 - \alpha$.

2.4 Evaluation of performance of var models

The methodology used in this study to assess the performance of VaR models assumes counting the number of times actual losses exceed the estimates resulting from VaR methodology.

The result of this count in a given period requires the consideration of a certain level of confidence to estimate the model.

Considering the number of daily logarithmic returns, R_{PF} , and the number of forecasts calculated by VaR, for a given confidence level (VaR^α) and for $P(R_{PF,t+1} < -VaR_{t+1}^\alpha) = \alpha$, a binary sequence is obtained, also called “Hit sequence”, given the number of overtakings of VaR_{t+1}^α , as follows:

$$I_t = \begin{cases} 1, & \text{se } R_{PF,t+1} < -VaR_{t+1}^\alpha \\ 0, & \text{se } R_{PF,t+1} \geq -VaR_{t+1}^\alpha \end{cases} \quad (25)$$

The “Hit sequence” shows the value 1, the day $t + 1$, if the loss on that day exceeds the VaR forecasted in advance for that day. If the VaR is not exceeded, then the sequence takes the value 0.

From the “Hit sequence” Backtesting of VaR is applied. Among the major tests available, we point out the test of unconditional coverage, independence test and conditional coverage test, which will be discussed in the following paragraphs.

2.4.1 Unconditional coverage test or Kupiec test

The unconditional coverage test or Kupiec (1995) test involves counting the number of times the VaR estimates produced are exceeded by an asset or a portfolio of assets, from a given sample. The null hypothesis of this test establishes that the true proportion of exceptions, π , is in line with the quantile of failures, α , provided by the VaR model:

$$H_0 : E[I_t] \equiv \pi = \alpha \quad (26)$$

The maximum likelihood test statistic for this hypothesis is:

$$LR_{uc} = -2 \ln \left(\frac{\alpha^{T_1} (1 - \alpha)^{T_0}}{\hat{\pi}^{T_1} (1 - \hat{\pi})^{T_0}} \right) \sim \chi^2(1) \quad (27)$$

in which T_1 is the number of failures ($I_t = 1$), for a given total number of days $T(I_t)$, T_0 is the number of successes ($I_t = 0$) and $\hat{\pi}$ (T_1/T) corresponds to the proportion of failures (exceptions).

2.4.2 Independence test

Christoffersen (2003) developed the independence test to examine the occurrence of exceptions in cluster, testing the null hypothesis of exceptions in the model being independent and identically distributed (IID), i.e., that the probability of exceptions occurring is equal, regardless an exception has occurred the day before. The test intends to examine how the exceptions occur, assuming that any two elements of the sequence must be independent of each other. When this condition is not met, it is an indicator that the VaR model is not sensitive enough to accommodate changes in market risk (CAMPBELL, 2005).

The independence test therefore provides the statistical conditions that allow one to reject the VaR model, except when in the cluster. For this purpose, it is assumed that the “hit sequence” is dependent over time and can be described by a first-order Markov sequence.

The Independence hypothesis ($\pi_0 = \pi_1$) can be tested using the likelihood ratio test

$$LR_{ind} = -2 \ln \left[L(\hat{\pi}) / L(\hat{\Pi}_1) \right] \sim \chi^2_1 \quad (28)$$

in which $L(\hat{\pi})$ is the likelihood, under the alternative hypothesis, based on the LR_{uc} test.

The statistical test LR_{ind} , to the independence of the number of consecutive exceptions, in t and $t - 1$, is

$$LR_{ind} = -2 \ln \left(\frac{(1-\hat{\pi})^{T_{00}+T_{10}} \hat{\pi}^{T_{10}+T_{11}}}{(1-\hat{\pi}_{01})^{T_{00}} \hat{\pi}_{01}^{T_{01}} (1-\hat{\pi}_{11})^{T_{10}} \hat{\pi}_{11}^{T_{11}}} \right) \sim \chi^2(1) \quad (29)$$

In small sample sizes, nullity of T_1 is frequent. In such cases, the LR_{ind} statistic is calculated by

$$LR_{ind} = -2 \ln \left(\frac{(1-\hat{\pi})^{T_{00}+T_{10}} \hat{\pi}^{T_{10}+T_{11}}}{(1-\hat{\pi}_{01})^{T_{00}} \hat{\pi}_{01}^{T_{01}}} \right) \sim \chi^2(1) \quad (30)$$

in which $T_j, i, j = 0,1$ is the number of observations, with j to follow i . The probability of an exception happening tomorrow, considering that none has happened today, is given by $\hat{\pi}_{01} = \frac{T_{01}}{T_{00}+T_{01}}$, whereas the probability of a conditional exception taking place tomorrow, as today an exception occurred, is given by $\hat{\pi}_{11} = \frac{T_{11}}{T_{10}+T_{11}}$. In turn, π concerns the failure rate or exceptions noted.

2.4.3 Conditional coverage test or Christoffersen test

To test the unconditional coverage and independence, Christoffersen (2003) developed the statistics LR_{uc} and LR_{ind} . To simultaneously test both properties, the same author developed the conditional coverage test, LR_{cc} , which is given by

$$LR_{cc} = LR_{ind} + LR_{uc} \sim \chi^2(2) \quad (31)$$

3 DATA AND EMPIRICAL RESULTS

3.1 Data and statistics

In order to analyze the performance of market risk management models, market indexes representing the international markets were selected – namely European, non-European, developed and emerging market indexes –, according to the classification used by Morgan Stanley Capital International, and which, in 2010, accounted for about 62% of world stock market capitalization, as seen in Table 1. The set of developed markets include European and non-European markets. From the European continent, the markets of Germany (DAX 30), France (CAC 40), the UK (FTSE 100), Spain (IBEX 35), Ireland (ISEQ Overall), Greece (ATG) and Portugal (20 PSI) were selected. Of the set of non-European developed markets, the USA (Dow Jones), Japan (Nikkei 225) and Hong Kong (Hang-Seng) were chosen. As regards emerging markets, Brazil (Bovespa) and India (Sensex) were selected.



TABLE 1 – Stock market capitalization, in percentage of world capitalization

USA	UK	France	Japan	Spain	Brazil	Germany	Portugal	Greece	Hong Kong	India	Ireland
30.5	5.5	3.4	7.3	2.1	2.8	2.5	0.1	0.1	4.8	2.9	0.06

Source: World Bank (c2014)

The data used in this study was obtained from Econostats and cover the period from October 4, 1999 to June 30, 2011, in which two major crises in the stock markets took place, the crisis of technology companies and the global financial crisis. The time gap between these two crises (2003-2007) corresponded to the period of a general rise in values in global stock indexes.

The series of closing values of the indexes were transformed into return series, r_t , by means of the application of $\ln(P_t/P_{t-1})$, in which P_t and P_{t-1} represent the closing values of a particular index on days t and $t-1$, respectively.

Table 2 presents the main descriptive statistics of each index return series, the Jarque-Bera tests of the normality of the series, the stationarity tests and the LM tests of Engle (1982) to the presence of heteroskedasticity.

The analysis of descriptive statistics allows us to draw the conclusion that only six of the 12 indexes had positive daily average return.

All series of returns, without exception, show signs of deviation from the normality hypothesis, as the coefficients of asymmetry and kurtosis are statistically different from a normal distribution (0 and 3, respectively). To confirm whether the fitting of the normal distribution is appropriate to the empirical distributions of the 12 series in all study periods, we also applied the Jarque-Bera backtesting. Taking into account the respective associated probabilities (equal to 0), we concluded that all series are statistically significant at 1%, clearly rejecting the hypothesis of normality thereof.

TABLE 2 – Descriptive statistics of the series

	Mean	Maximum	Minimum	Standard deviation	Asymmetry	kurtosis	JB (Prob.)	ADF (Prob.)	LM (Prob.)
ATG	-0.000513	0.088129	-0.10214	0.016668	-0.154716	6.9734	(0.0000)	(0.0000)	(0.0000)
BOV	0.000597	0.136766	-0.145659	0.019319	-0.223056	7.6515	(0.0000)	(0.0000)	(0.0000)
CAC	-0.00005	0.105946	-0.094715	0.015649	0.042147	7.9632	(0.0000)	(0.0000)	(0.0000)
DAX	0.000119	0.107975	-0.074335	0.016158	0.061832	7.2076	(0.0000)	(0.0000)	(0.0000)
DJ	0.000061	0.105083	-0.082005	0.012625	0.031963	10.6597	(0.0000)	(0.0000)	(0.0000)
FTSE	-0.000006	0.093842	-0.092646	0.013034	-0.105256	9.1214	(0.0000)	(0.0000)	(0.0000)
HANG	0.000191	0.134073	-0.135825	0.016327	-0.007715	10.848	(0.0000)	(0.0000)	(0.0000)
IBEX	0.000026	0.134836	-0.108341	0.015264	0.047132	9.5441	(0.0000)	(0.0000)	(0.0000)
ISEQ	-0.000166	0.097331	-0.139636	0.01498	-0.652159	11.0664	(0.0000)	(0.0000)	(0.0000)
NIKKEI	-0.000205	0.094941	-0.12111	0.015462	-0.644815	9.0135	(0.0000)	(0.0000)	(0.0000)
PSI	-0.000113	0.101959	-0.103792	0.011677	-0.223954	12.9958	(0.0000)	(0.0000)	(0.0000)
SENSEX	0.000483	0.159901	-0.118092	0.017047	-0.207255	9.6325	(0.0000)	(0.0000)	(0.0000)

Source: Author.

In order to determine the non-stationarity or integration of the series, we applied the traditional ADF test. The null hypothesis of the test states that the series have a unit root, i.e., that the series are integrated of order 1, versus the alternative hypothesis that the series do not have a unit root. The results confirm that the values of the probabilities of the tests of the 12 series do not exceed 1%, and clearly rejects the null hypothesis of integration of the series, and concludes that they show stationary or are I (0).

To test the existence of conditional heteroskedasticity (ARCH effects), the rates of return of the indexes, autoregressive processes of first order were estimated, and LM tests of Engle (1982) were applied, to lag 20, residuals of autoregressive processes. In all cases, the probabilities of the LM tests allow us to conclude that, for a significance level of 1%, the series of rates of return indexes exhibit conditional heteroskedasticity, so the use of GARCH-type models proves adequate.

3.2 Empirical results

To assess the performance of VaR models in capturing the risk of the 12 markets studied,

we considered a theoretical investment portfolio, equally weighted, from which various multivariate conditional heteroskedasticity models were estimated, namely the VECH, BEKK and CCC specifications, for normal t-student distributions, and the asymmetrical effect, and on which VaR estimates were based. Finally, the EVT was applied in the three specifications for GARCH and Threshold GARCH (TGARCH) models, in order to incorporate the asymmetrical effect, and the normal and t-student distributions, in order to understand if it responds adequately to extreme variances that characterized the period studied. In all such cases, the estimates of VaR models considered confidence levels of 5%, 1% and 0.5%. The first confidence level follows the RiskMetrics methodology, the second on takes into account the requirements of the Basel II Committee, whereas the third one was chosen in order to realize the consequence of a more demanding estimate condition.

Table 3 compares total VaR of the various indexes with the portfolio's VaR for the different estimate models and for the three confidence levels.

TABLE 3 – Comparison between portfolio var and total VaR estimates of indexes

Distribution		Model						
Normal		Vech	Tvech	BEKK	TBEKK	CCC	TCCC	
Confidence level	0.005	Total VaR	0.0396	0.0396	0.0396	0.0395	0.0397	0.0395
		Portfolio VaR	0.0279	0.0279	0.0279	0.0278	0.0280	0.0278
	0.01	Total VaR	0.0358	0.0358	0.0357	0.0357	0.0358	0.0357
		Portfolio VaR	0.0252	0.0252	0.0252	0.0251	0.0253	0.0251
	0.05	Total VaR	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263
		Portfolio VaR	0.0178	0.0178	0.0178	0.0178	0.0179	0.0178
t-student		Vech	Tvech	BEKK	TBEKK	CCC	TCCC	
Confidence level	0.005	Total VaR	0.0437	0.0437	0.0437	0.0442	0.0437	0.0440
		Portfolio VaR	0.0308	0.0308	0.0308	0.0311	0.0308	0.0310
	0.01	Total VaR	0.0381	0.0381	0.0381	0.0384	0.0381	0.0384
		Portfolio VaR	0.0269	0.0269	0.0268	0.0270	0.0269	0.0270
	0.05	Total VaR	0.0255	0.0254	0.0254	0.0254	0.0255	0.0256
		Portfolio VaR	0.0176	0.0176	0.0176	0.0176	0.0176	0.0177

Notes: VaR estimates presented in the table were obtained from the variance-covariance matrix of GARCH models, without asymmetric effect (VECH, BEKK and CCC) and asymmetric effect (TVECH, TBEKK and TCC), considering the confidence intervals of 99.5% (0.5) 99% (1) 95% (5).

Source: Author.

In all cases compared, the VaR of the portfolio is clearly lower than total individual VaR's indexes. Therefore, there seems to be a reason to believe that market risk can be mitigated by means of diversification. Although the markets tend to linked closer and closer, the option for an investment strategy is based on the assumption of international diversification, which considers a broad set of markets, may constitute a form of protection against market risk.

Table 4 presents the mean of VaR estimates of the various models. The most obvious feature is that the mean values depend more on the

distribution considered than on the forecast of the covariance matrix. In most cases, the mean VaR estimates, produced based on the t-student distribution, exceed those resulting from normal distribution, especially in the confidence levels of 0.5% and 1%. Only VECH and TVECH-EVT models, these two confidence levels, and the TCCC-EVT model, the confidence level of 0.5%, do not support this superiority. For the highest quantile, the conclusion is contrary, revealing superiority of estimates based on the normal distribution, in 67% of cases.

TABLE 4 – Mean estimates of VaR models

	Normal			t-student		
	0.005	0.01	0.05	0.005	0.01	0.05
VECH	0.02522	0.02277	0.01610	0.02439	0.02127	0.01395
BEKK	0.02534	0.02289	0.01618	0.02745	0.02370	0.01521
CCC	0.02458	0.02220	0.01570	0.02675	0.02333	0.01530
TVECH	0.02482	0.02241	0.01585	0.03122	0.02647	0.01636
TBEKK	0.02499	0.02257	0.01596	0.02656	0.02306	0.01498
TCCC	0.02411	0.02178	0.01540	0.02676	0.02333	0.01530
VECH-TVE	0.03092	0.02522	0.01572	0.03289	0.02635	0.01575
BECK-TVE	0.03409	0.02705	0.01580	0.03546	0.02766	0.01555
CCC-TVE	0.03139	0.02542	0.01557	0.03179	0.02574	0.01577
TVECH-TVE	0.03200	0.02590	0.01585	0.03189	0.02580	0.01576
TBEKK-TVE	0.03068	0.02520	0.01597	0.03381	0.02677	0.01556
TCCC-TVE	0.03048	0.02482	0.01539	0.03047	0.02494	0.01566

Notes: VaR estimates presented in the table were obtained from the variance-covariance matrix of GARCH models, without asymmetric effect (VECH, BEKK, CCC) and asymmetric effect (TVECH, TBEKK and TCC) and EVT, considering the confidence intervals of 99.5% (0.5) 99% (1) 95% (5).

Source: Author.

Table 5 summarizes the failure rate of various estimate models, i.e., the percentage of times the VaR estimates were exceeded by daily returns as a result of the unexpected market events. The results presented in this table reveal that the models that do not resort to extreme value theory, whether on the assumption of normal distribution or t-student distribution, generally showed difficulties in adapting to extreme changes in market.

In the case of estimates based on GARCH-type models, only the highest level of confidence, in the normal distribution, produces a failure rate

that does not exceed the respective confidence level. On the other hand, this is not true for any event in the t-student distribution, which is why one can conclude that their estimates are too conservative for various confidence levels. In general, the simple fact of the asymmetric effect does not significantly improve the previous situation. The incorporation of the asymmetric effect specification, however, has the VECH model record a positive change in the three quantiles, with the percentage of exceptions to be, in all cases, lower than their confidence levels.

The use of the extreme value theory leads to different conclusions from those listed above, about the multivariate GARCH models. Estimates of GARCH models and TGARCH models, based on the two distributions, provide identical conclusions, with percentages of

exceptions below levels of confidence of 0.5% and 1%, contrary to what happens to the confidence level of 5%. This fact gives a first indication of the superiority of the extreme value theory compared to the other methods, as we shall confirm with the implementation of a performance test.

TABLE 5 – Failure rate of VaR models

	Normal			t-student		
	0.005	0.01	0.05	0.005	0.01	0.05
VECH	0.00898	0.01485	0.04696	0.01209	0.02383	0.06457
BEKK	0.01174	0.01865	0.04696	0.00863	0.01623	0.05387
CCC	0.00932	0.01588	0.04938	0.00760	0.01278	0.05283
TVECH	0.00967	0.01588	0.04903	0.00311	0.00725	0.04523
TBEKK	0.00898	0.01519	0.05007	0.00829	0.01657	0.05559
TCCC	0.00967	0.01588	0.05007	0.00656	0.01140	0.05283
VECH-TVE	0.00276	0.00898	0.05007	0.00345	0.00829	0.05007
BECK-TVE	0.00311	0.00794	0.05007	0.00276	0.00760	0.05007
CCC-TVE	0.00311	0.00829	0.05007	0.00345	0.00794	0.05007
TVECH-TVE	0.00345	0.00863	0.05007	0.00345	0.00863	0.05007
TBEKK-TVE	0.00345	0.00863	0.05007	0.00311	0.00829	0.05007
TCCC-TVE	0.00276	0.00794	0.05007	0.00345	0.00794	0.05007

Notes: VaR estimates presented in the table were obtained from the variance-covariance matrix of GARCH models, without asymmetric effect (VECH, BEKK, CCC) and asymmetric effect (TVECH, TBEKK and TCC) and EVT, considering the confidence intervals of 99.5% (0.5) 99% (1) 95% (5).

Source: Author.

After estimating the different VaR models, Backtesting procedures were applied in order to reach a conclusion on the ability of these models to effectively manage the market risk of the stock markets. Firstly, we considered the unconditional coverage test or Kupiec test, which is a development of the failure rate, set by the Basel Committee, to oversee Banking activities. Secondly, the independence test was applied to test whether exceptions are IID. Finally, we applied the conditional coverage test, or Christoffersen test. *Backtesting* results are presented in tables 6 to 13.

In the unconditional coverage test and for two smaller quantiles, the t-student distribution showed better performance than the normal distribution. In both distributions, the

performance of the estimates improved at the highest level of confidence, with global acceptance levels of the models of 100% and 92% in the normal and t-Student distributions, respectively. Models that did not resort to EVT, the TCCC model, based on the t-student distribution, showed the best performance, being accepted for all confidence levels. In contrast, the VECH model (t-student) was rejected for the three quantiles. As for the models based on EVT, all passed the Kupiec test, the three levels of confidence. The specifications of the variance-covariance matrix did not reject the null hypothesis of the test, and supported the idea of the superiority of the methodology based on the generalized Pareto distribution, given the methodologies based on the other two distributions.

TABLE 6 – Backtesting results - GARCH model and normal distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2870	2853	2760	2862	2842	2760	2869	2850	2753
T_1	26	43	136	34	54	136	27	46	143
T_{00}	2844	2810	2636	2830	2793	2642	2842	2805	2625
T_{01}	26	43	124	32	49	118	27	45	128
T_{10}	26	43	124	32	49	118	27	45	128
T_{11}	0	0	12	2	5	18	0	1	15
p	0.009	0.015	0.047	0.012	0.019	0.047	0.009	0.016	0.049
π_{01}	0.009	0.015	0.045	0.011	0.017	0.043	0.009	0.016	0.046
π_{11}	0.000	0.000	0.088	0.059	0.093	0.132	0.000	0.022	0.105
LR_{uc}	7.443 (0.006)	5.983 (0.014)	0.574 (0.449)	19.137 (0.000)	17.431 (0.000)	0.574 (0.449)	8.660 (0.003)	8.592 (0.003)	0.024 (0.878)
LR_{ind}	0.471 (0.492)	1.296 (0.255)	4.415 (0.036)	18.613 (0.000)	8.663 (0.003)	20.192 (0.000)	0.508 (0.476)	0.092 (0.761)	7.696 (0.006)
LR_{cc}	7.914 (0.019)	7.280 (0.026)	4.989 (0.083)	37.750 (0.000)	26.093 (0.000)	20.766 (0.000)	9.168 (0.010)	8.684 (0.013)	7.719 (0.021)

Notes: This table presents the results of the unconditional coverage (LR_{uc}), independence (LR_{ind}) and conditional coverage (LR_{cc}) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the GARCH-VECH GARCH-BEKK and GARCH-CCC models, assuming that returns are described by normal distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 7 – Backtesting results - GARCH model and t-student distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2861	2827	2709	2871	2849	2740	2874	2859	2743
T_1	35	69	187	25	47	156	22	37	153
T_{00}	2828	2762	2547	2847	2807	2607	2852	2824	2606
T_{01}	33	65	162	24	42	133	22	35	137
T_{10}	33	65	162	24	42	133	22	35	137
T_{11}	2	4	25	1	5	23	0	2	16
π	0.012	0.024	0.065	0.009	0.016	0.054	0.008	0.013	0.053
π_{01}	0.012	0.023	0.060	0.008	0.015	0.049	0.008	0.012	0.050
π_{11}	0.057	0.058	0.134	0.040	0.106	0.147	0.000	0.054	0.105
LR_{uc}	20.887 (0.000)	40.292 (0.000)	11.903 (0.001)	6.304 (0.012)	9.552 (0.002)	0.890 (0.345)	3.384 (0.066)	2.073 (0.150)	0.480 (0.488)
LR_{ind}	3.207 (0.073)	2.570 (0.109)	12.562 (0.000)	1.549 (0.213)	11.136 (0.001)	20.192 (0.000)	0.337 (0.562)	2.845 (0.092)	6.921 (0.009)
LR_{cc}	24.094 (0.000)	42.862 (0.000)	24.465 (0.000)	7.852 (0.020)	20.687 (0.000)	21.082 (0.000)	3.721 (0.156)	4.918 (0.086)	7.402 (0.025)

Notes: This table presents the results of the unconditional coverage (LR_{uc}), independence (LR_{ind}) and conditional coverage (LR_{cc}) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the GARCH-VECH GARCH-BEKK and GARCH-CCC models, assuming that returns are described by t-Student distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 8 – Backtesting results - TGARCH model and normal distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2868	2850	2754	2870	2852	2751	2868	2850	2751
T_1	28	46	142	26	44	145	28	46	145
T_{00}	2840	2804	2626	2844	2809	2624	2840	2804	2619
T_{01}	28	46	128	26	43	127	28	46	132
T_{10}	28	46	128	26	43	127	28	46	132
T_{11}	0	0	14	0	1	18	0	0	13
π	0.010	0.016	0.049	0.009	0.015	0.050	0.010	0.016	0.050
π_{01}	0.010	0.016	0.046	0.009	0.015	0.046	0.010	0.016	0.048
π_{11}	0.000	0.000	0.099	0.000	0.023	0.124	0.000	0.000	0.090
LR_{uc}	9.952 (0.002)	8.592 (0.003)	0.057 (0.811)	7.443 (0.006)	6.807 (0.009)	0.000 (0.986)	9.952 (0.002)	8.592 (0.003)	0.000 (0.986)
LR_{ind}	0.547 (0.460)	1.485 (0.223)	6.249 (0.012)	0.471 (0.492)	0.148 (0.701)	12.972 (0.000)	0.547 (0.460)	1.485 (0.223)	4.165 (0.041)
LR_{cc}	10.499 (0.005)	10.077 (0.006)	6.306 (0.043)	7.914 (0.019)	6.955 (0.031)	12.972 (0.002)	10.499 (0.005)	10.077 (0.006)	4.165 (0.125)

Notes: This table presents the results of the unconditional coverage (LRuc), independence (LRind) and conditional coverage (LRcc) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the Threshold GARCH (VECH, BEKK and CCC) models, assuming that returns are described by normal distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 9 – Backtesting results - TGARCH model and t-student distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2887	2875	2765	2872	2848	2735	2877	2863	2743
T_1	9	21	131	24	48	161	19	33	153
T_{00}	2878	2854	2645	2848	2805	2596	2858	2830	2606
T_{01}	9	21	120	24	43	139	19	33	137
T_{10}	9	21	120	24	43	139	19	33	137
T_{11}	0	0	11	0	5	22	0	0	16
π	0.003	0.007	0.045	0.008	0.017	0.056	0.007	0.011	0.053
π_{01}	0.003	0.007	0.043	0.008	0.015	0.051	0.007	0.012	0.050
π_{11}	0.000	0.000	0.084	0.000	0.104	0.137	0.000	0.000	0.105
LR_{uc}	2.411 (0.121)	2.444 (0.118)	1.428 (0.232)	5.245 (0.022)	10.554 (0.001)	1.844 (0.174)	1.291 (0.256)	0.545 (0.460)	0.480 (0.488)
LR_{ind}	0.056 (0.813)	0.307 (0.580)	3.888 (0.049)	0.401 (0.527)	10.751 (0.001)	15.844 (0.000)	0.251 (0.616)	0.761 (0.383)	6.921 (0.009)
LR_{cc}	2.467 (0.291)	2.750 (0.253)	5.316 (0.070)	5.646 (0.059)	21.305 (0.000)	17.688 (0.000)	1.542 (0.463)	1.306 (0.521)	7.402 (0.025)

Notes: This table presents the results of the unconditional coverage (LRuc), independence (LRind) and conditional coverage (LRcc) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the Threshold GARCH (VECH, BEKK and CCC) models, assuming that returns are described by t-Student distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 10 – Backtesting results – GARCH-EVT model and normal distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2888	2870	2751	2887	2873	2751	2887	2872	2751
T_1	8	26	145	9	23	145	9	24	145
T_{00}	2880	2844	2621	2878	2850	2625	2878	2848	2622
T_{01}	8	26	130	9	23	126	9	24	129
T_{10}	8	26	130	9	23	126	9	24	129
T_{11}	0	0	15	0	0	19	0	0	16
π	0.003	0.009	0.050	0.003	0.008	0.050	0.003	0.008	0.050
π_{01}	0.003	0.009	0.047	0.003	0.008	0.046	0.003	0.008	0.047
π_{11}	0.000	0.000	0.103	0.000	0.000	0.131	0.000	0.000	0.110
LR_{uc}	3.481 (0.062)	0.316 (0.574)	0.000 (0.986)	2.411 (0.121)	1.333 (0.248)	0.000 (0.986)	2.411 (0.121)	0.911 (0.340)	0.000 (0.986)
LR_{ind}	0.044 (0.833)	0.471 (0.492)	7.200 (0.007)	0.056 (0.813)	0.368 (0.544)	15.192 (0.000)	0.056 (0.813)	0.401 (0.527)	8.969 (0.003)
LR_{cc}	3.526 (0.172)	0.788 (0.675)	7.200 (0.027)	2.467 (0.291)	1.701 (0.427)	15.192 (0.001)	2.467 (0.291)	1.312 (0.519)	8.970 (0.011)

Notes: This table presents the results of the unconditional coverage (LR_{uc}), independence (LR_{ind}) and conditional coverage (LR_{cc}) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the GARCH (VECH, BEKK and CCC) and EVT models, assuming that returns are described by normal distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 11 – Backtesting results – GARCH-EVT model and t-student distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2886	2872	2751	2888	2874	2751	2886	2873	2751
T_1	10	24	145	8	22	145	10	23	145
T_{00}	2876	2848	2624	2880	2852	2626	2876	2850	2622
T_{01}	10	24	127	8	22	125	10	23	129
T_{10}	10	24	127	8	22	125	10	23	129
T_{11}	0	0	18	0	0	20	0	0	16
p	0.003	0.008	0.050	0.003	0.008	0.050	0.003	0.008	0.050
π_{01}	0.003	0.008	0.046	0.003	0.008	0.045	0.003	0.008	0.047
π_{11}	0.000	0.000	0.124	0.000	0.000	0.138	0.000	0.000	0.110
LR_{uc}	1.563 (0.211)	0.911 (0.340)	0.000 (0.986)	3.481 (0.062)	1.842 (0.175)	0.000 (0.986)	1.563 (0.211)	1.333 (0.248)	0.000 (0.986)
LR_{ind}	0.069 (0.792)	0.401 (0.527)	12.972 (0.000)	0.044 (0.833)	0.337 (0.562)	17.549 (0.000)	0.069 (0.792)	0.368 (0.544)	8.969 (0.003)
LR_{cc}	1.633 (0.442)	1.312 (0.519)	12.972 (0.002)	3.526 (0.172)	2.179 (0.336)	17.549 (0.000)	1.633 (0.442)	1.701 (0.427)	8.970 (0.011)

Notes: This table presents the results of the unconditional coverage (LR_{uc}), independence (LR_{ind}) and conditional coverage (LR_{cc}) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the GARCH (VECH, BEKK and CCC) and EVT models, assuming that returns are described by t-Student distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 12 – Backtesting results – TGARCH-EVT model and normal distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2886	2871	2751	2886	2871	2751	2888	2873	2751
T_1	10	25	145	10	25	145	8	23	145
T_{00}	2876	2846	2618	2876	2846	2624	2880	2850	2619
T_{01}	10	25	133	10	25	127	8	23	132
T_{10}	10	25	133	10	25	127	8	23	132
T_{11}	0	0	12	0	0	18	0	0	13
π	0.003	0.009	0.050	0.003	0.009	0.050	0.003	0.008	0.050
π_{01}	0.003	0.009	0.048	0.003	0.009	0.046	0.003	0.008	0.048
π_{11}	0.000	0.000	0.083	0.000	0.000	0.124	0.000	0.000	0.090
LR_{uc}	1.563 (0.211)	0.573 (0.449)	0.000 (0.986)	1.563 (0.211)	0.573 (0.449)	0.000 (0.986)	3.481 (0.062)	1.333 (0.248)	0.000 (0.986)
LR_{ind}	0.069 (0.792)	0.435 (0.509)	2.919 (0.088)	0.069 (0.792)	0.435 (0.509)	12.972 (0.000)	0.044 (0.833)	0.368 (0.544)	4.165 (0.041)
LR_{cc}	1.633 (0.442)	1.009 (0.604)	2.920 (0.232)	1.633 (0.442)	1.009 (0.604)	12.972 (0.002)	3.526 (0.172)	1.701 (0.427)	4.165 (0.125)

Notes: This table presents the results of the unconditional coverage (LRuc), independence (LRind) and conditional coverage (LRcc) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the Threshold GARCH (VECH, BEKK and CCC) and EVT models, assuming that returns are described by normal distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

TABLE 13 – Backtesting results – TGARCH-EVT model and t-student distribution assumption.

	VECH (0.5%)	VECH (1%)	VECH (5%)	BEKK (0.5%)	BEKK (1%)	BEKK (5%)	CCC (0.5%)	CCC (1%)	CCC (5%)
T_0	2886	2871	2751	2887	2872	2751	2886	2873	2751
T_1	10	25	145	9	24	145	10	23	145
T_{00}	2876	2846	2618	2878	2848	2624	2876	2850	2621
T_{01}	10	25	133	9	24	127	10	23	130
T_{10}	10	25	133	9	24	127	10	23	130
T_{11}	0	0	12	0	0	18	0	0	15
π	0.003	0.009	0.050	0.003	0.008	0.050	0.003	0.008	0.050
π_{01}	0.003	0.009	0.048	0.003	0.008	0.046	0.003	0.008	0.047
π_{11}	0.000	0.000	0.083	0.000	0.000	0.124	0.000	0.000	0.103
LR_{uc}	1.563 (0.211)	0.573 (0.449)	0.000 (0.986)	2.411 (0.121)	0.911 (0.340)	0.000 (0.986)	1.563 (0.211)	1.333 (0.248)	0.000 (0.986)
LR_{ind}	0.069 (0.792)	0.435 (0.509)	2.919 (0.088)	0.056 (0.813)	0.401 (0.527)	12.972 (0.000)	0.069 (0.792)	0.368 (0.544)	7.200 (0.007)
LR_{cc}	1.633 (0.442)	1.009 (0.604)	2.920 (0.232)	2.467 (0.291)	1.312 (0.519)	12.972 (0.002)	1.633 (0.442)	1.701 (0.427)	7.200 (0.027)

Notes: This table presents the results of the unconditional coverage (LRuc), independence (LRind) and conditional coverage (LRcc) tests of a portfolio comprising 12 indexes and VaR estimates, obtained from the variance-covariance matrix of the Threshold GARCH (VECH, BEKK and CCC) and EVT models, assuming that returns are described by t-Student distribution and considering confidence intervals of 99.5% (0.5), 99% (1) and 95% (5). The values in brackets refer to the probability of each test.

Source: Author.

The analysis of the temporal independence of exceptions revealed that the models generally reported good performance in the two lower quantiles. Only the BEKK model evidenced difficulty detecting exceptions in clusters, in particular in the normal distribution. Regarding the highest quantile, the models showed clear difficulty in identifying independence, and the TVECH-EVT model was the only one to prove the ability to capture the effect of independence, patent in both distributions. On the other hand, the comparison between both distributions translates into very similar results, with the normal distribution to validate 91.7%, 91.7% and 8.3% of models in quantiles 0.5%, 1% and 5%, respectively, whereas the t-Student distribution contributes to validate 100%, 83.3% and 8.3% of the models in each of said quantiles. These results allow the conclusion that none of the distributions has superiority over the other, and the models ability to test property of exceptions that are IID does not depend on the type of distribution considered.

The conditional coverage test, which combines the effects of unconditional coverage and independence, evidenced the superiority of models based on the t-student distribution compared to those based on the normal distribution, at the confidence levels of 0.5% and 1%, with acceptance at 83% and 75% of the cases for the two previous confidence levels, respectively, vis-à-vis the 50% acceptance in the normal distribution. For the highest level of confidence, the situation changes with the normal distribution (33.3%), which is higher than the t-student (16.7%), although in both cases the demonstrated performance has been very limited.

Several different models proved their inability as risk management instruments, at the three confidence levels, in particular models with the assumption of normal distribution (BEKK, CCC, and TVECH and TBEKK). This also happened with some models supported by the t-student distribution, as was the case

of VECH and BEKK models. Conversely, the TVECH (t-student), TVECH-EVT (normal and t-student) and TCCC-EVT (normal) models revealed consistent performances in the three quantiles. We should point out that the EVT led to a marked improvement in the performance of VaR models, especially in the lower two quantiles, with all the models inspired by this methodology passing the Christoffersen test. Regarding the highest confidence level, only TVECH and TCCC models showed good performance, the first in both distributions, and the second on in the normal distribution.

Results suggest that the most important element in the accuracy of estimates of VaR models is the use of EVT, whose theoretical genesis uses the generalized Pareto distribution. This situation highlights the importance of this approach, as a determining factor in the performance of the models, similar to results obtained by other authors – for example, Lopez and Walter (2001); in a second order of importance, the specification of the multivariate model appears.

Of the set of specifications considered in this study, the BEKK multivariate model clearly presented the worst performance, while the TVECH-EVT model stood out positively among the others, and was validated in all tests and in both distributions, revealing therefore its ability to incorporate the extreme conditions experienced in the markets, which marked the period considered in this study, as a consequence of the Dot-Com and global financial crises.

4 CONCLUSION AND FUTURE IMPLICATIONS

In this paper, we studied the suitability of VaR estimates to market conditions, considering a theoretical investment portfolio, comprising 12 international stock market indexes for the period between the Dot-Com crisis and the global financial crisis. To this end, we started with

the covariance matrixes, generated by various multivariate conditional heteroskedasticity models, estimated in accordance with the normal and t-Student distributions, to predict the VaR, similarly to the methodologies used in other papers. Differently with respect to these papers, however, we suggested a new methodology that combines these models with the extreme values theory, as a VaR estimate methodology, in order to try to conveniently accommodate the high turbulence that characterized stock markets, including those representing European states under international financial assistance.

The comparison between the total VaR's of the various indexes and the VaR of the investment portfolio allowed the conclusion that, in all events, the portfolio VaR is clearly inferior. This fact allows the opportunity to minimize market risk, when an investor bets on a strategy of international diversification. Although the link between stock markets is growing closer, a diversification strategy that considers a broad set of markets can be a form of investor protection against market risk.

The application of performance validation tests of VaR estimates reveals that the most important element in the accuracy of these estimates is the use of EVT, based on the generalized Pareto distribution. Then, in terms of importance, is the specification of the multivariate model. Of the set of specifications considered in this study, the BEKK multivariate model was clearly the worst in performance, whereas the TVECH-EVT model stood out positively from the others, and was validated in all tests and in both distributions. It revealed its ability to incorporate extreme market conditions experienced during the time period studied.

The results show that the methodology proposed in this paper can accommodate the high turbulence in the markets, and it can be seen as an appropriate option in the management of market risk.

In future work, it may be interesting to use an optimization model that considers, at the same level, market returns and risk estimates, and relate the multivariate GARCH to the EVT

models, in order to obtain additional information to international investors on their investment alternatives, taking into account the opportunities offered by emerging and developed markets.

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